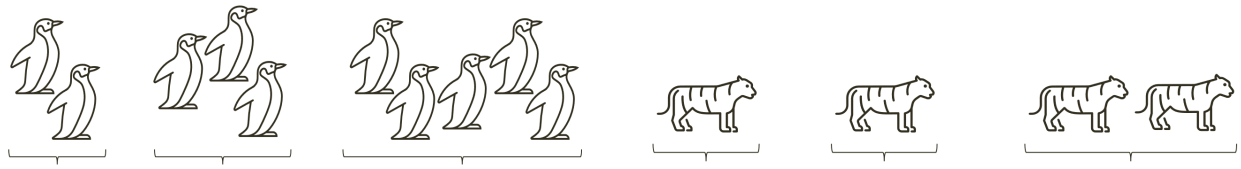
A Solution to the Problem of Minimal Parts (Outline/Handout Draft 12/12)

## 1 Background

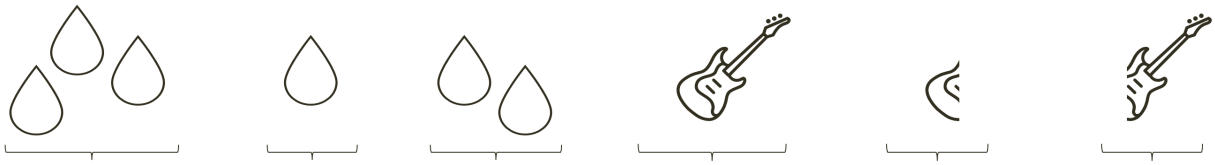
When you make reference to something plural or mass, you are also referring to the parts which are also plural or mass. We want to capture this.

Some nouns in English are homogeneous. That is, if you combine two portions of the stuff to which the noun refers, the result can also be referred to with that noun—this is called cumulativity. The same goes for dividing a portion referred to by that noun into two portions—this is called divisibility.

Plural and non-countable nouns share the feature of **homogeneous reference**, which involves **cumulativity** and **divisiveness**



*Penguins* and *penguins* are *penguins*, but *a tiger* and *a tiger* is not *a tiger*.



*Water* divided is *water* and *water*, but *a guitar* is not *a guitar* and *a guitar*.

## 2 The Problem

This can be formalized as the semantic properties of cumulative and divisive reference to distinguish between singular, plural, and mass noun reference (Bunt 1985; Krifka 1989; Link 1983; Quine 1960). Note cf. Rothstein (2010) p. 351 it is crucial to define divisiveness with at least two parts that do not overlap, otherwise something can be trivially divisive. Divisibility is also called distributivity in the literature; these terms seem pretty interchangeable. [1]

(1) P is cumulative iff: $\forall x \forall y[P(x) \wedge P(y) \rightarrow P(x \cup y)]$
P has cumulative reference if when x and y are P, then the sum of x and y is also P.

(2) P is divisive iff: $\forall x \forall y[P(x) \wedge y \subseteq x \rightarrow P(y)]$
P has divisive reference if any part of P is also P.

If a noun's reference is both cumulative and divisive, it is homogeneous. However, many have noted that divisive reference cannot hold true of a noun like *water* since it contains parts that are not themselves water—the hydrogen and oxygen atoms forming each molecule. If a glass of water contains a billion water

[1] Definitions taken from Krifka (1989) and Rothstein (2010).

molecules, the water could be divided into two smaller portions which would have some 500 million water molecules each, but would still clearly classify as *water*. This process can be repeated over and over again, halving the number of molecules in each portion each time, but still ending up with a glass containing *water*. However, if this is repeated to the point at which there is only one molecule left in the glass, one would have to split the $H_2O$ molecule into hydrogen and oxygen atoms; of the resulting substances, none of them can be referred to accurately with the noun *water*.

## 3 Previous Proposals

**3.1 Reference.** One way to address the Problem of Minimal Parts is to argue that language does not refer directly to the structure of the world (Gillon 1992; Parsons 1970; Quine 1960).

If the referent of *water* is a mental concept then there are no issues with ascribing homogeneous reference to some nouns even when the thing in the world does have parts. For instance, Quine (1960) argues that the distinction between general and singular terms is a feature of the terms (presumably, of the grammar) and not a feature of "the stuff they name" (p. 91). Gillon (1992) adopts an approach which he calls the "weak homogeneous reference hypothesis" that "grammar is simply mute on the question of whether or not there are minimal parts" (p. 598). (also I think Parsons (1970) and Landman (2010) for sure fall into this group, Link (1983) somewhat though he just sidesteps the issue).

- Pros: no more Problem of Minimal Parts
- Cons: requires some separation between reference and structure of the referent, does not capture the similar pattern of reference for non-countable and plural nouns
- Predictions: divisive nouns like *water* will refer to any amount of water, even when the referent is a single $H_2O$ molecule

**3.2 Vagueness.** The point at which reference picks out minimal parts is vague for non-countable nouns (Cai (2015) and Chierchia (2010, 2017), see also Liebesman (2016) for an argument against Chierchia).

- Pros: no more Problem of Minimal Parts, vagueness is elsewhere in language
- Cons: it's vague (Liebesman 2016), does not capture the similar pattern of reference for non-countable and plural nouns
- Predictions: speakers will never refer to the minimal parts of non-countable nouns

**3.3 Granularity.** Divisive reference only holds above a certain level of granularity (Champollion 2015, 2017). he calls this "approximate" divisive reference (Champollion 2017, p. 49)

Champollion (2015) proposes a formal bounded/unbounded distinction. "Stratified reference requires a predicate that holds of a certain entity or event to also hold of its parts along a certain dimension down to a certain level of granularity. Dimension and granularity are understood as parameters which different distributive constructions can set to different values" (p. 110-111).

"In order to model different levels of granularity, I will assume that there is a vague predicate $\varepsilon$ that takes a set $K$ of type $\langle \alpha, t \rangle$ and an entity $\chi$ of type $\beta$, where $\alpha$ and $\beta$ range over at least the following basic

types: temporal intervals, spatial extents, degrees, and numbers. I assume that $\varepsilon(K)(\chi)$ holds just in case $\chi$ counts as very small as compared to the comparison class $K$" (p. 118).

- Pros: no more Problem of Minimal Parts
- Cons: needs to postulate level of granularity where divisiveness stops for each case
- Predictions: no reference once the referent is smaller than the specified level

## 4 Proposal: Discard Homogeneity

If homogeneity has so many issues, why is it worth keeping it around? (Rothstein 2010) [might move this up into the first proposal and/or re-divide these up]

- Pros: gets rid of the problem
- Cons: we need another proposal (which comes with its own baggage)
- Predictions: depends on what the other proposal is, but just rejection of homogeneity would lead to no predictions

## 5 Proposal: Redefine Parthood

Instead of discarding divisiveness, another approach would be to redefine it in a way that can accurately capture the data. If the way that a portion of some *water* is 'a part' is different from the way that hydrogen and oxygen atoms are 'parts' of *water* then we can define a divisiveness that applies to one and not the other.

- Pros: gets rid of the problem, fits with some intuitions about what it means to be a part, explains why divisiveness for *furniture* doesn't split up a chair, better captures the behavior of plurals (cf. Wągiel (2021) who argues that we need non-atomic approaches to handle count singular and plurals)
- Cons: we have to redefine terms, introduces a more complex view of part
- Predictions: anything with (clearly accessible) structured parts would be countable on this view, so things like individual molecules and atoms, or pieces of a larger object-mass collective

**5.1 Defining Parthood by Composition.** Fine (2010) presents four principles which distinguish the different grounds upon which identity of a composed object can hold. While these are technically not rules or restrictions on composition, they are principles that distinguish valid forms of composition and the resulting thing. Thus, they generate different sorts of parthood relationships and different sorts of wholes. Following Fine, these are formulated as identity statements, rather than as definitions or parthood relations. This might at first seem odd but it nicely captures the intuitions behind cumulativity and divisibility—that some stuff and some stuff together are the same as some stuff, and that part of a thing is equivalent to that thing. Fine uses the sum operator, $\sum$ to denote whatever results from composition. The following three definitions are the relevant ones from his work (the fourth, permutation, does not seem to pick out anything relevant to countability or homogeneity).

(3) Leveling: $\sum(\sum(w,x),\sum(y,z) = \sum(w,x,y,z))$
The embedding of components is irrelevant to the identity of the whole

(4) Absorption: $\sum(x,x,y,y) = \sum(x,y)$
The repetition of components is irrelevant to the identity of the whole

(5) Collapse: $\sum(x) = x$
The whole composed of a single component is identical to that component

(6) Permutation: $\sum(x,y,z) = \sum(y,z,x)$
The order of the components is irrelevant to the identity of the whole

I'm not quite sure how to (or if I should) rewrite these to be principles of reference directly, rather than identity of the referent, but the gist is that we're interested in these identity statements in the situations where the stuff on both sides can be referred to with the same noun phrase. So they're in some ways principles about what variation can exist between referents of a certain noun phrase (and I think the NP part is crucial here, because I think there's also ways to work out coercion, and plurality, in terms of allowing or restricting the reference of a noun according to these principles, so the parallel is between morphosyntax and identity principles, but also probably still something lexical... I digress).

Absorption is the principle upon which count use and non-count use differ. Non-countable nouns compose according to absorption, while count plurals resist absorption, e.g. $\sum(water, water) = \sum(water)$ but $\sum(apple, apple) \neq \sum(apple)$. In some sense this is the inverse of ways of talking about count nouns as having some 'unit for individuation' and instead talking about non-countable nouns as having the property of absorption with regards to reference.

Leveling distinguishes singular count uses from plural count and mass uses; this is the principle of identity which is (loosely) equivalent to homogeneity, e.g. $\sum(\sum(apple, apple), \sum(apple, apple)) = \sum(apple, apple, apple, apple)$, where both sides are appropriately referred to with *apples*. And having this distinction between absorption (countable/non-countable) and leveling (singular/multiple) rather than trying to find one principle to hang the distinction between singular AND plural count nouns vs. non-countable nouns on feels like it matches up better with the intuitions that count plurals share something in common with non-countable nouns but also something in common with singular count nouns.

Collapse is interesting, because singular count uses and mass uses both allow for collapse, but plural count (and, I think, object-mass nouns and collectives) do not—a single apple cannot be the referent of *apples* but it can be for *apple*, any portion of water can be the referent of *water*, but no single piece of furniture can be the referent of *furniture* or a single individual the referent of *congress*. In some sense this enforces that there's something about the 'whole' that a non-collapse-able noun refers to that is more than just itself and/or one of its parts. Collapse is loosely the equivalent of what is teased out with distributive predicates, I think—with nouns that are collapsed the predicate just applies to the whole thing, but with ones that cannot be collapsed, and have some internal structure, the predicates distribute. (I also think that Fine's fourth principle, permutation, might be relevant to the case of object mass nouns and collectives, but that's a topic for a different day).

The advantage that these principles give us—beyond a richer understanding of parthood and countability—

is a way to avoid the problem of minimal parts without positing an arbitrary level for each noun and/or context. Leveling can only apply where there are multiple parts (e.g. *apples* in a plurality of apples or *water* in a portion of water). Leveling cannot be a useful principle of identity resulting from the (de)composition of less than two things. For example, if we have two water molecules in a cup, leveling holds, as $\sum(\sum(water), \sum(water)) = \sum(water, water)$. But if we have one water molecule, leveling does not apply to *water* by definition of the principle. In fact, leveling and absorption both require multiple components, thus by definition when only one water molecule exists in our glass it cannot be referred to by a use of a noun that involves absorption or leveling; so the remaining option is a noun that does not compose on those principles—which would be a singular count noun.

## 6 Some Relevant Data

Speakers use count nouns in the contexts where the referents are not divisible, such as in the sentences in (7) for elements and in (8) for some molecules and compounds. The contexts that generate this type of reference are very specialized, but in all cases a count noun is used to refer to individual atoms or molecules. Despite this infrequency, in all cases a count noun is used to refer to individual atoms or molecules; this cannot be dismissed as a case of mass-count coercion—the nouns refer to objects, not standard portions or types.

(7) a. The two *arsenics* have a collective charge of plus six, and each *arsenic* has a charge of plus three.
b. Calcium chloride has two *chlorines* for each calcium.
c. (CH3CH2)4P2O7 molecules contain 8 *carbons*, 20 *hydrogens*, two *phosphoruses*, and seven *oxygens*.
d. As non-bridging *oxygens* are surrounded with *calciums* this energy is initially decreased until the *calciums* begin to crowd each other.
e. If we draw lines through the *titaniums*, every fifth one is missing.

(8) a. Maltose is created by condensation reaction of the two *glucoses*, forming a $\alpha$-1,4-O-glycosidic linkage.
b. Ice XI is a proton-ordered form of ice $I_h$, where *waters* orient in a repeated manner rather than the more typical random fashion.
c. Two *ozones* can combine to form three *oxygens*.
d. Since there are fewer methanol molecules close to the bilayer than there are *ethanols* the average area per methanol is larger than the average area per *ethanol*.
e. The chains can be straight or branched, and they can run to thousands of *sugars* in a single chain.

Speakers are shifting between mass and count uses of a noun depending on whether the referent in context is an un-individuated substance or an individual atom or molecule (Bunt 1985; Cheng 1973). If a speaker is referring to something they know is not divisible, they switch to COUNT reference; their use of nouns with homogeneous reference is selective. Minimal parts do not pose a problem as there simply are no contexts where mass nouns are being used at this level of reference.

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